

# **A Schumpeterian Model of Entrepreneurship, Innovation, and Regional Economic Growth<sup>1</sup>**

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## Abstract

We provide the first theoretical analysis of a one sector, discrete time, Schumpeterian model of growth in a regional economy in which consumers are risk neutral, there is no population growth, monopolistic entrepreneurs produce intermediate goods, and a single consumption good is produced competitively. Our analysis generates several new results. In the deterministic model, R&D in time  $t$  *surely* leads to an innovation in time  $t+1$ . In this setting, we show that relative to the balanced growth path (BGP) equilibrium, the social planner *always* allocates more labor to R&D and hence achieves a *larger* size of innovation and a *higher* growth rate. Next, in the stochastic model, R&D in time  $t$  *probabilistically* leads to an innovation in time  $t+1$ . In this setting, we first define the equilibrium and the steady state BGP allocations. Second, we generalize the notion of the steady state and determine the number of unemployed workers. Third, we show that our regional economy experiences bursts of unemployment followed by periods of full employment. Finally, we show that a decline in the time discount rate increases the average growth rate and the average unemployment.

**Keywords:** Creative Destruction, Dynamic, Entrepreneur, Innovation, Regional Economy, Stochastic

**JEL Codes:** R10, E10, O31

## 1. Introduction

Few topics in the past three decades have evoked as much research interest among economists as has the topic of economic growth. In addition, with the passage of time, researchers have increasingly focused on the *region* as a salient unit of analysis and they have now convincingly shown that regional growth and development are very closely related to the activities of competing entrepreneurs. In the words of Fischer and Nijkamp (2009, p. 184), “[e]ntrepreneurship...is central to regional economic development.” In addition, contend Fischer and Nijkamp (2009, p. 183), an “entrepreneurial culture is a prerequisite for the wealth of regions...”

Present day research on entrepreneurship in the context of regional economic growth and development has emphasized three noteworthy points.<sup>4</sup> First, entrepreneurship involves an intertemporal process in which new firms are being created, existing firms are growing, and unsuccessful firms are winding down their operations. Second, entrepreneurship involves control of this intertemporal process by the entrepreneur/owner who also serves as a manager of risk. Finally, entrepreneurship entails innovation in a probabilistic and competitive market environment.

In addition to emphasizing the above three points, the sizeable but mainly empirical and case study based literature on the trinity of entrepreneurship, innovation, and regional economic growth has addressed a number of other pertinent issues. For instance, Lorentzen (2008) has used network theory and empirical results to argue that firms find knowledge sources on different spatial scales and that global networks and knowledge sources are very beneficial to them. After pointing out that innovations and the capacity to innovate are crucial factors for regional development, Cornett (2009)

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See O’Farrell (1986), Malecki (1997), Verheul *et al.* (2002), Fischer *et al.* (2006), and Fischer and Nijkamp (2009) for a more detailed corroboration of this claim.

focuses on Denmark and studies the factors that facilitate regional growth and the mechanisms that stimulate innovative behavior in large, small, and medium sized enterprises. Michael and Pearce (2009) note that through innovation, entrepreneurship creates wealth for both individuals and nations. Therefore, these authors contend that it makes sense for governments to support entrepreneurship because by doing so, these same governments are actually encouraging innovation. Nijkamp (2009) pays particular attention to what he calls the “regional action space of entrepreneurs” and then he surveys the nexuses between entrepreneurship and regional economic growth.

Schwartz and Gothner (2009) empirically analyze the extent to which business incubators have been successful in promoting entrepreneurship, innovation, and regional economic development. Valliere and Peterson (2009) use data from the Global Entrepreneurship Monitor and the Global Competitiveness Report and show that in developed nations, a significant fraction of the economic growth rates can be attributed to high expectation entrepreneurs who exploit national investments in knowledge creation and regulatory freedom. Focusing on the links between knowledge and innovation, Vaz and Nijkamp (2009) develop a “knowledge circuit model” that takes into account all the pertinent stakeholders and then offers a framework for research in applied policy problems. Finally, Henderson and Weiler (2010) first sketch the salient relationships between entrepreneurship, innovation, and economic growth and then they empirically assess the relationship between entrepreneurship and job growth across United States labor market areas and counties.

The papers discussed in the preceding two paragraphs have certainly enhanced our understanding of the many nexuses between the trinity of entrepreneurship, innovation, and regional economic growth. This notwithstanding, to the best of our knowledge, there are virtually *no*

*theoretical* studies that are both dynamic and stochastic in nature and that study the ways in which entrepreneurship and innovation endogenously influence economic growth in a regional economy. Given this state of affairs, in our paper, we analyze *deterministic* and *stochastic* versions of a stylized one sector Schumpeterian model of growth in a regional economy in discrete time in which consumers are risk neutral, there is no population growth, monopolistic entrepreneurs produce intermediate goods of distinct qualities, and a single consumption good is produced competitively.

More specifically, in the deterministic version of the model, R&D in time  $t$  *necessarily* leads to a vertical, process innovation in time  $t+1$ . In this setting, we first characterize the balanced growth path (BGP) for our regional economy and then we compare the BGP growth rate with the Pareto optimal growth rate. Next, in the stochastic version of the model, R&D in time  $t$  *probabilistically* leads to a vertical, process innovation in time  $t+1$ . In this setting, we undertake four analytical tasks. First, we define the equilibrium and the steady state BGP allocations. Second, we generalize the notion of the steady state and determine the number of unemployed workers. Third, we show that our regional economy experiences bursts of unemployment followed by periods of full employment. Finally, we point out that a decline in the time discount rate increases the average growth rate and the average unemployment in our regional economy.

The rest of this paper is organized as follows. Section 2.1 first describes the deterministic version of the one sector Schumpeterian model that is adapted from the prior work of Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992). Section 2.2 describes the BGP and then specifies restrictions on the parameters so that the pertinent transversality condition is satisfied. Section 2.3 compares the BGP growth rate with the Pareto optimal growth rate and shows that the quality or size of innovations is always too small relative to the size of innovations in the Pareto

optimal allocation. Section 3.1 delineates the stochastic version of the section 2.1 model. Section 3.2 through 3.5 provide a detailed discussion of the four analytical tasks described in the previous paragraph. Section 4 concludes and then discusses potential extensions of the research delineated in this paper.

## **2. The Deterministic Schumpeterian Model**

### ***2.1. Preliminaries***

We begin by focusing on a stylized regional economy that is subject to the forces of innovation. To this end, consider an infinite horizon regional economy in which only a single sector, that is the subject of study in this paper, experiences quality improvements over time. Time is discrete. Since we are interested in working with a model of endogenous technology, firms and individuals in our regional economy must ultimately have a choice between different kinds of technologies and, in this regard, greater effort, investment, or research spending ought to lead to the invention of better technologies. These features tell us that there must exist a *meta production function* or a “production function over production functions” which tells us how new technologies are generated as function of various *inputs*. Following Acemoglu (2009, p. 413), we shall refer to this meta production function as the “innovation possibilities frontier.” It is important to note that we are using the term “inputs” in a very general sense. In other words, these inputs can be intermediate goods, machines, or even capital. With this caveat in mind, in the remainder of this paper, we shall refer to these inputs as “intermediate goods.”

Consumers in our regional economy are risk neutral and their constant relative risk aversion (CRRA) utility function in time  $t$  is  $\{C(t)^{1-\theta}-1\}/(1-\theta)$ ,  $\theta \neq 1$ , where  $C(t)$  is consumption in time  $t$ ,

and  $\theta$  is the constant coefficient of relative risk aversion.<sup>5</sup> In what follows, we shall abstract away from issues related to population growth in our regional economy. The consumption good sector in our regional economy has the production function

$$Y(t) = \frac{1}{1-\beta} x(t/q)^{1-\beta} \{q(t)L_E(t)\}^\beta, \quad (1)$$

where  $Y(t)$  is the output of the consumption good,  $q(t)$  is the *quality* of the *unique* intermediate good used in production,  $x(t/q)$  is the *quantity* of this intermediate good used in time  $t$ ,  $\beta$  is a parameter, and  $L_E(t)$  is the amount of labor used in the production of the consumption good in time  $t$ .

The total endowment of labor (workers) in our regional economy is  $L$  and  $L \geq L_E(t) + L_R(t)$ . In other words, in any time  $t$ , workers in this economy can work either in the consumption good sector  $\{L_E(t)\}$  or in the R&D sector  $\{L_R(t)\}$  where entrepreneurs with research firms engage in innovative activities and thereby attempt to *invent* new intermediate goods. An entrepreneur—with a research firm—who successfully generates an innovation can use this innovation to effectively monopolize the intermediate good sector. Note that this “monopolistic entrepreneur” remains a monopolist only until the appearance of the next innovation. Put differently, we can think of this monopolist as having acquired a patent on the new intermediate good that lasts for one time period. There is a linear technology for the production of the intermediate goods so that an intermediate good, once invented, can be produced at the constant marginal cost  $\psi$  in terms of the consumption good. The use of this intermediate good as an input permits more efficient methods to be used in the production of the consumption good (see equation (1)).

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See Acemoglu (2009, p. 308) for more on the properties of CRRA utility functions.

The “engine” of regional economic growth in our paper is *process innovations* that lead to quality improvements. As such, the R&D technology in our regional economy is such that the use of  $L_R > 0$  workers in time  $t$  necessarily leads to an innovation in time  $t+1$  and the number of workers used in research determines the quality or the size of the innovation via the function  $\Lambda(L_R)$ . Put differently, if date  $t$  quality of an intermediate good is  $q$  then the new date  $t+1$  intermediate good has quality  $q' = \Lambda(L_R)q$ . Now, to stress the salience of labor use in the R&D sector for the occurrence of innovations, we suppose that innovations in our regional economy occur only when  $L_R > 0$ . In addition, for mathematical tractability, we also suppose that the function  $\Lambda(\cdot)$  is strictly increasing, differentiable, strictly concave, and that it satisfies the so called Inada conditions.<sup>6</sup> With this background in place, the task for us now is to characterize the balanced growth path (BGP) for our regional economy. In the course of doing so, we shall specify restrictions on the parameters of our model so that the pertinent transversality condition is satisfied.

## 2.2. The equilibrium and the BGP

As a precursor to characterizing the BGP, let us first describe an equilibrium in our regional economy. To this end, given the current quality  $q$  of the intermediate good, let  $p^x(t/q)$ ,  $r(t/q)$ , and  $w(t/q)$  denote at time  $t$ , the price of the intermediate good, the interest rate, and the wage, respectively. Now, given the current quality and a deterministic path for this quality  $[q(t)]_p$ , an equilibrium is a collection of time paths or trajectories of allocations and prices

$$[Y(t/q), C(t/q), x(t/q), L_E(t/q), L_R(t/q), p^x(t/q), r(t/q), w(t/q)]_{t=0}^{\infty} \quad (2)$$

with the property that the representative consumer maximizes utility, the consumption good sector maximizes profits given prices, the monopolistic intermediate good producer chooses quantities and

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See Acemoglu (2009, pp. 33-34) for more on the Inada conditions.



prices to maximize profits, the R&D sector hires researchers to maximize profits, and all markets clear.

The representative consumer's optimization problem gives us the Euler equation

$$C(t/q)^{-\theta} = (1+r)\exp(-\rho)C(t+1)^{-\theta}, \quad (3)$$

where  $\rho$  is the time discount rate, and the transversality condition

$$\lim_{t \rightarrow \infty} \exp(-\rho t) C(t)^{-\theta} V(t/q) = 0, \quad (4)$$

where  $V(t/q)$  is a value function that represents the net present discounted value of owning the blueprint of an intermediate good of quality  $q$  in time  $t$ . Since the representative consumer is risk neutral, we can tell that the coefficient of relative risk aversion  $\theta=0$ . Hence the Euler equation (equation (3)) is satisfied if and only if the interest rate is the inverse of the time discount rate. In symbols, the pertinent condition is

$$1+r(t/q) = \exp(\rho). \quad (5)$$

An obvious implication of equation (5) is that the interest rate is *constant* in equilibrium.

The consumption good producers' maximization problem gives us the following demand function for intermediate goods

$$x(t/q) = q(t) L_E(t) p^x(t/q)^{-1/\beta}. \quad (6)$$

Now, to reduce the number of cases that we need to study and also to clearly bring out the key Schumpeterian aspect of our model, we suppose that once a new intermediate good is invented, the old vintage is destroyed through obsolescence—this is the Schumpeterian *creative destruction* in our model—and hence the new “monopolistic entrepreneur” faces no competition from previous incumbents and he can price his intermediate good at the unconstrained monopoly price.<sup>7</sup> Since the

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In the language of Aghion and Howitt (1992, p. 328), the innovations we are studying in this paper are “drastic” innovations.

monopolistic entrepreneur or the intermediate good producer faces a demand function that is isoelastic, his pricing decision satisfies

$$p^x(t/q) = \frac{1}{1-\beta} \Psi = 1, \quad (7)$$

which also tells us that  $x(t/q) = qL_E(t)$  and that per period profits are  $\pi(q) = \beta qL_E(t)$ . Knowing these last two expressions, we can tell that wages are now given by

$$w(t/q) = \frac{\beta}{1-\beta} \frac{x(t/q)^{1-\beta} \{q(t)L_E(t)\}^\beta}{L_E(t)} = \frac{\beta}{1-\beta} q, \quad (8)$$

and the output of the consumption good sector is given by

$$Y(t/q) = \frac{1}{1-\beta} qL_E(t/q). \quad (9)$$

In this deterministic Schumpeterian model, the current monopolistic entrepreneur gets replaced by a different monopolistic entrepreneur in the next time period with *certainty*. Therefore, the present monopolistic entrepreneur's value function is only the period profits and this is given by

$$V(t/q) = \pi(t/q) = \beta qL_E(t/q). \quad (10)$$

Given current quality  $q$ , the R&D sector solves

$$\Pi_R(t/q) = \max_{\{L_R\}} \frac{1}{1+r} V\{t+1/\Lambda(L_R)q\} - L_R w(t/q) = \max_{\{L_R\}} \frac{1}{1+r} \beta \Lambda(L_R) q L_E(t+1/q) - L_R w(t/q). \quad (11)$$

The first order necessary condition for an optimum to the above maximization problem is

$$w(t/q) \geq \frac{1}{1+r} \beta \Lambda' \{L_R(t/q)\} q L_E(t+1/q), \text{ with equality if } L_R(t/q) > 0. \quad (12)$$

The reader should note that in our deterministic Schumpeterian model, the R&D sector makes profits in equilibrium.<sup>8</sup> As such, we suppose that the shares of these R&D firms are held equally by the consumers in our regional economy so that profits accrue to the representative consumer.

Having described the equilibrium for our regional economy, we are now in a position to delineate a BGP equilibrium in which the allocation of labor is constant over time, i.e.,  $L_E(t) \equiv L_E$  and  $L_R(t) \equiv L_R$ , for all  $t$ . Since the innovation function  $\Lambda(L_R)$  satisfies the Inada conditions, equation (12) will always have an interior solution and, in addition, a particular condition is satisfied. Using equations (5), (8), and the labor market clearing condition  $L_E + L_R = L$ , we can write this particular condition as

$$(1 - \beta) \Lambda'(L_R) (L - L_R) = \exp(\rho). \quad (13)$$

Equation (13) tells us that in the regional economy under study, the BGP allocation of labor to the R&D sector or  $L_R$  depends on the monopoly markup, the time discount rate, and the R&D technology. In particular, this BGP allocation does *not* depend on the quality of the existing intermediate good, because, on the one hand, higher quality intermediate goods generate more profits but, on the other hand, higher quality intermediate goods raise wages and hence make additional innovations costlier.

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For an alternate model in which the R&D sector is characterized by free entry and hence makes zero profits in equilibrium, see Acemoglu (2009, pp. 468-472).

The BGP equilibrium is completely characterized by equation (13). In fact, once we determine the values of  $L_R$  and  $L_E$ , output (of the consumption good) is determined by equation (9) and consumption is given by the net output. In other words,

$$C(t/q) = Y(t/q) - (1 - \beta)x(t/q) = \left[ \frac{1}{1 - \beta} - (1 - \beta) \right] q L_E. \quad (14)$$

In each time period, the quality, and therefore output, consumption, and wages, all grow by the factor of  $\Lambda(L_R)$ . The transversality condition—see equation (4)—in our model will be satisfied as long as

$$\lim_{t \rightarrow \infty} \exp(-\rho t) C(0) \Lambda(L_R)^t = 0. \quad (15)$$

From the above discussion it follows that the constant growth path we have described is an equilibrium with positive growth whenever the parametric condition  $0 < \ln\{\Lambda(L_R)\} < \rho$  is satisfied. This completes the discussion of the equilibrium and the BGP equilibrium for our regional economy. We now proceed to compare the BGP growth rate with the Pareto optimal growth rate for our regional economy.

### 2.3. The BGP and the Pareto optimal growth rates

Let us first compute the optimal choice of intermediate good production by the social planner given that the quality of the intermediate good is  $q$  and that employment in the production of the consumption good is  $L_E(t)$ . For this *static* problem, the social planner solves

$$\max_{\{x\}} C(t/q) = \frac{1}{1 - \beta} x^{1 - \beta} \{q L_E(t)\}^\beta - (1 - \beta)x, \quad (16)$$

which tells us that

$$x(t/q)=qL_E(t)(1-\beta)^{-1/\beta} \quad (17)$$

and that

$$C(t/q)=\beta(1-\beta)^{-1/\beta}qL_E(t). \quad (18)$$

Now, let us analyze the dynamic tradeoff for the social planner and then determine the allocation of labor between the R&D and the other sectors. The social planner's dynamic optimization problem is

$$\max_{\{L_R(t), L_E(t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \exp(-\rho t) C(t), \quad (19)$$

subject to

$$C(t)=\beta(1-\beta)^{-1/\beta}q(t)L_E(t), \quad (20)$$

$$q(t+1)=q(t)\Lambda\{L_R(t)\}, \quad (21)$$

and

$$L_R(t)+L_E(t)=L, \quad \forall t \geq 0. \quad (22)$$

The first order necessary condition for an optimum for the choice variable  $L_R(t)$  is

$$\beta(1-\beta)^{-1/\beta}q(t)=\Lambda'\{L_R(t)\}q(t)\exp(-\rho)\beta(1-\beta)^{-1/\beta}L_E(t+1), \quad \forall t \geq 0. \quad (23)$$

Let us conjecture a solution to the above first order necessary condition in which  $L_E \equiv L_E(t)$  and  $L_R \equiv L_R(t)$  are constant for all time  $t$ . Using this conjecture, the above first order necessary condition simplifies to

$$\Lambda'(L_R)(L-L_R)=\exp(\rho), \quad (24)$$

which has a unique solution and hence our conjecture is verified. Note that our social planner's optimization problem is weakly concave in the relevant choice variables. Therefore, it follows that the conjectured path that satisfies the first order conditions is indeed optimal if the transversality

condition  $\ln \Lambda(L_R) < \rho$  holds. Then, the social planner's allocation of workers in the R&D sector is also constant and given by the solution to equation (24). Further, quality, output, and consumption, all grow by the factor of  $\Lambda(L_R)$ .

Recall that the function  $\Lambda(\cdot)$  is a strictly decreasing function. Using this piece of information along with equations (13) and (24), we conclude that

$$L_R^S > L_R^{Eq} \Rightarrow \Lambda(L_R^S) > \Lambda(L_R^{Eq}). \quad (25)$$

The above two results tell us that the social planner *always* allocates more labor to R&D and hence achieves a *larger* size of innovation and a *higher* growth rate. The reason for this outcome is the following. The social planner's static allocation is unaffected by monopoly distortions that are captured by the  $(1 - \beta)$  term in equation (13) and that is absent in equation (24). Therefore, the social planner produces more intermediate goods for a given quality level. This means that every unit of quality that is innovated is more valuable to the social planner than to an equilibrium firm which, in turn, implies that the social planner innovates more and achieves a higher growth rate. This concludes our comparative discussion of the BGP and the Pareto optimal growth rates in our regional economy. We now move on and conduct a detailed analysis of the stochastic version of the basic, deterministic model of this section.

### 3. The Stochastic Schumpeterian Model

#### 3.1. Preliminaries

The one sector Schumpeterian model of this section is essentially the same as the one we analyzed in section 2. However, there is one key difference. Now, the function  $\Lambda(\cdot)$  denotes the *probability* of innovation. Each innovation improves an intermediate good's quality  $q$  to  $\lambda q$  where  $\lambda > 1$ . Further, when a new innovation arrives a fraction  $\phi$  of workers employed in the consumption good

sector are unable to adapt to this new technology and hence become unemployed for one time period. During this period, these unemployed workers “retrain” or “retool” themselves. Our task now is to define the equilibrium and the steady state BGP allocations for this stochastic Schumpeterian model.

Recall from the analysis in section 2 that risk neutrality on the part of consumers in our regional economy implies that the interest rate is given by

$$r(t)=r=\exp(\rho)-1, \quad (26)$$

and that the equilibrium wages and profits are given by

$$w(t/q)=\frac{\beta}{1-\beta}q(t), \quad (27)$$

and

$$\pi(t/q)=\beta q(t)L_E(t). \quad (28)$$

The basic new feature in the model of this section is the labor market. The specification of technology in the first paragraph of this section tells us that capturing this technological progress is the only way to generate growth in our model. However, this feature also imposes challenges on our regional economy in the short run. We can think of these challenges as either changes in the sectoral composition of labor or as the required skill level of the workforce. In what follows, we shall capture these features in a straightforward manner by simply supposing that a fraction  $\varphi$  of the workers employed in the consumption good sector will be unemployed in order to get retrained. Let us denote unemployed workers by  $L_U(t)$  so that total labor  $L=L_R(t)+L_E(t)+L_U(t)$ .

To capture the retraining aspect of our model, we will need to define and work with a new state variable. To this end, let  $\zeta(t) \in \{U, E\}$  denote the *state* of the economy. That is, we shall denote

a state where there has been no innovation in the last period by  $E$  (full employment) and a state where an innovation occurred in the last time period by  $U$  (unemployment). Mathematically, we thus have

$$\left. \begin{array}{l} U \text{ if } q(t) > q(t-1) \\ E \text{ if } q(t) = q(t-1) \end{array} \right\} = \zeta(t). \quad (29)$$

Using the notation in equation (29), we can express the number of unemployed people as

$$\left. \begin{array}{l} \varphi\{L - L_R(t)\} \text{ if } \zeta(t) = U \\ 0 \text{ if } \zeta(t) = E \end{array} \right\} = L_U(t). \quad (30)$$

The timing of the various activities in our stochastic model is as follows. At time  $t$  labor is allocated according to  $L_R(t)$ ,  $L_U(t)$ , and  $L_E(t)$ . With probability  $\Lambda\{L_R(t)\}$  there is an innovation in time  $t$  so that tomorrow's quality is given by  $q(t+1) = \lambda q(t)$ . In this case, some people in the labor force allocated to work in the consumption good sector will be unemployed as they have to learn to work with the new intermediate good of higher quality. If there is no innovation then we have  $q(t+1) = q(t)$  and all workers allocated to the consumption good sector can be used for production. In other words,  $L_E(t+1) = L - L_R(t+1)$ .

Let  $X(t)$  denote the total spending on intermediate goods in our regional economy. Then, an equilibrium in our regional economy now consists of time paths or trajectories of allocations  $[Y(t), C(t), X(t), q(t), L_E(t), L_R(t)]_{t=0}^{\infty}$ , a value function  $V(t/q, \zeta)$ , and prices  $[p^x(t/q, \zeta), w(q), r(t)]_{t=0}^{\infty}$  such that the representative consumer maximizes utility taking prices as given, the final (consumption) good producers maximize profits at given prices, the monopolistic entrepreneur



producing the intermediate good maximizes profits, the R&D sector hires the optimal amount of labor (researchers) given the value function, and all markets clear. A BGP allocation is an allocation where the flow rate or probability of innovation is *constant*. Note that in our regional economy, this does *not* mean that all variables grow at a constant rate. Along the BGP, the probability of an innovation is constant but its actual occurrence is still a random variable. Finally, note that both the value function and intermediate good prices are functions of the state variable  $\zeta$ , but wages are not. This completes our description of the equilibrium and the steady state BGP allocations for the stochastic Schumpeterian model under study in this section.

### 3.2. *The steady state and unemployed workers*

We now wish to generalize the notion of the steady state and then determine the number of unemployed workers in our regional economy. To this end, let us first solve the model described in section 3.1. First, we need to find a value function to characterize the equilibrium behavior of the research firms. This naturally leads to the following question. What is the value of being the monopolistic entrepreneur with an intermediate good of quality  $q$ ? This entrepreneur's current profits are given by equation (28). In addition, Ken Arrow's so called "replacement effect" implies that the current incumbent (monopolistic entrepreneur) will *not* be active in research.<sup>9</sup> Therefore, from this incumbent's point of view, the probability that there is an innovation in time  $t$  is the same as the probability that this incumbent will be replaced in the next time period. Now, denoting the replacement probability for an incumbent with quality  $q$  in state  $\zeta$  in time  $t$  by  $p(t, q, \zeta)$ , the value of being a monopolistic entrepreneur is given by the functional equations

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The "replacement effect" refers to the idea that a monopolist has lower incentives to undertake innovation than does a competitive firm because the innovation will *replace* its own extant profits. See Arrow (1962) and Acemoglu (2009, pp. 420-421) for more on this effect.

$$V(t,q,U)=\pi(t,q,U)+\frac{1}{1+r}\{1-p(t,q,U)\}V(t+1,q,E) \quad (31)$$

and

$$V(t,q,E)=\pi(t,q,E)+\frac{1}{1+r}\{1-p(t,q,E)\}V(t+1,q,E). \quad (32)$$

In writing equations (31) and (32), we have used the facts that the interest rate is constant and that the per period profits depend on the state variable  $\zeta$  via the available labor supply (also see equation (28)).

To understand why we need two functional equations to pin down the value of innovation, note that in the first period of being a monopolistic entrepreneur, the state of our regional economy is  $\zeta(t)=U$  since this monopolistic entrepreneur himself had the innovation in the last period. Hence, in his first period of using his innovation, the profits are lower since the economy is characterized by unemployment. In case this monopolistic entrepreneur does not get replaced (which happens with probability  $1-p(t,q,U)$ ), he remains the sole provider of the intermediate good, and he obtains a value  $V(t+1,q,E)$ . This captures the fact that the quality stays the same because the incumbent monopolistic entrepreneur does *not* engage in research and, conditional on survival, the regional economy's state is  $\zeta(t+1)=E$  as there was no innovation in time  $t$  (otherwise the monopolist would have been replaced). The value of being the monopolist in state  $E$  with quality  $q$  then consists of the per period profits  $\pi(t,q,E)$  and the continuation value  $V(t+1,q,E)$  which arises with probability  $1-p(t,q,E)$ .

Let us now describe the BGP allocation in our regional economy. In section 3.1, we have defined the BGP as an equilibrium in which the probability of an innovation is constant. The

probability of an innovation is essentially the probability of replacement and this is given by  $p(t, q, \zeta) = \Lambda\{L_R(t, q, \zeta)\}$ . Because this replacement probability is only a function of the number of employed researchers, for  $p(t, q, \zeta)$  to be constant, we need  $L_R(t, q, \zeta) = L_R^*$  for all  $t, q$ , and  $\zeta$ . Put differently, we need the number of researchers to be constant along a BGP. Using the market clearing condition for the labor market and equation (30), we can tell that the number of workers producing the consumption good and the number of unemployed workers are given by

$$L_E(t, q, E) = L_E(E) = L - L_R^*, \quad (33)$$

$$L_U(t, q, U) = L_U(U) = (1 - \varphi)(L - L_R^*), \quad (34)$$

$$L_U(t, q, E) = L_U(E) = 0, \quad (35)$$

and

$$L_U(t, q, U) = L_U(U) = \varphi(L - L_R^*). \quad (36)$$

Therefore, along a BGP, employment in production of the consumption good and unemployment are only a function of the state of the regional economy  $\zeta(t)$  but independent of time and the current quality  $q$  of the intermediate good. Using this fact and equation (28), we can solve for the BGP per period profits. We get

$$\pi(t, q, E) = \pi(q, E) = \beta q L_E(E) = \beta q (L - L_R^*), \quad (37)$$

and

$$\pi(t, q, U) = \pi(q, U) = \beta q L_U(U) = \beta q (1 - \varphi)(L - L_R^*). \quad (38)$$

Denoting the constant BGP innovation probability by  $p^{BGP}(t, q, \zeta) = \Lambda(L_R^*) = p^*$ , we see that the value functions in equations (31) and (32) are independent of time and functions only of the current quality  $q$  and the state of the economy  $\zeta$ . Therefore, the value of being a monopolistic entrepreneur can be written as

$$V(q,U)=\pi(q,U)+\frac{1}{1+r}(1-p^*)V(q,E), \quad (39)$$

and

$$V(q,E)=\pi(q,E)+\frac{1}{1+r}(1-p^*)V(q,E). \quad (40)$$

An implication of equation (40) is that  $V(q,E)=\{(1+r)/(r+p^*)\}\pi(q,E)=\{(1+r)/(r+p^*)\}\beta qL_E(E)$ .

Using this last expression along with equations (33) and (34), equation (39) can be rewritten as

$$V(q,U)=\beta qL_E(U)+\frac{1-p^*}{r+p^*}\beta qL_E(E)=(1-\phi+\frac{1-p^*}{r+p^*})\beta qL_E(E). \quad (41)$$

Given the value function in equation (41), research firms select the number of researchers  $L_R$  to solve

$$\max_{\{L_R\}} \Lambda(L_R) \frac{1}{1+r} V(\lambda q, U) - w(t) L_R. \quad (42)$$

To understand the above maximization problem, note that if a research firm employs  $L_R$  researchers, it achieves an innovation with probability  $\Lambda(L_R)$ . This innovation has a value of  $V(\lambda q, U)$  but this value materializes only in the next time period and hence it is discounted. However, the wage bill has to be paid for in the present time period. Now, the number of researchers hired satisfies the

following first order necessary condition<sup>10</sup>

$$\Lambda'(L_R) \frac{1}{1+r} (1 - \varphi + \frac{1+r}{r+p^*}) \beta \lambda q(t) L_E(E) = \frac{\beta}{1-\beta} q(t), \quad (43)$$

which, upon further simplification, reduces to

$$(1-\beta) \Lambda'(L_R) (1 - \varphi + \frac{1+r}{r+p^*}) \lambda L_E(E) = 1+r. \quad (44)$$

Along the BGP we have  $p^* = \Lambda(L_R^*)$  and  $L_E(E) = L - L_R^*$ . Using these two conditions, we can rewrite equation (44) as

$$(1-\beta) \Lambda'(L_R^*) (1 - \varphi + \frac{1+r}{r+\Lambda(L_R^*)}) \lambda (L - L_R^*) = 1+r. \quad (45)$$

Note that equation (45) determines the BGP number of researchers  $L_R^*$  as a function only of the parameters of the underlying problem. In particular and as required for the BGP,  $L_R^*$  is neither a function of time and nor is it a function of the current quality  $q(t)$ . Recall that by assumption  $\Lambda(\cdot)$  is a strictly concave function. Therefore, the left-hand-side (LHS) of equation (45) is strictly decreasing in  $L_R$ . In addition, we suppose that an Inada type condition for the innovation function, namely,  $\lim_{L \rightarrow \infty} \Lambda'(L) = \infty$ , holds. Then it follows that there exists a unique value  $L_R^*$  that solves equation (45).

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Because  $\Lambda(\cdot)$  is a strictly concave function, the following first order necessary condition is also sufficient to characterize the optimum of interest.

If we compare equation (45) with the corresponding condition in section 2, we see that the structure of the deterministic and the stochastic regional economies is very similar. In section 2, the key equation which pinned down allocation in the labor market was equation (13). Now, if we let the constant fraction of workers who are unemployed  $\phi=0$  and then we compare equations (13) and (45) then it is clear the only difference in the two equations stems from the product of the parameter  $\lambda$  and the discount rate  $(1+r)/\{r+\Lambda(L_R^*)\}$ . This last discount rate incorporates in it the fact that existing patents now expire with probability  $p^*=\Lambda(L_R^*)$  instead of with probability one which was implicit in the analysis in section 2. Our penultimate task is to show that in the probabilistic setting of this section, our regional economy experiences bursts of unemployment followed by periods of full employment.

### ***3.3 Unemployment and employment patterns***

We begin by noting that we have already characterized the dynamic behavior of unemployment in our regional economy in equation (30). In the discussion leading up to equation (30) in section 3.1, we noted that unemployment is positive whenever the labor force has to be retrained to adapt itself to the new technology and it is zero otherwise. However, in the one final good sector Schumpeterian model that we are studying in this section, innovations occur stochastically. Therefore, it is in this sense that our regional economy will feature bursts of unemployment followed by periods of full employment.

In particular, whenever a new innovation occurs—and this happens with probability  $\Lambda(L_R^*)$ —the regional economy under study experiences unemployment in the following period. In contrast, when no innovation takes place, all workers find jobs in the consumption good sector and the economy experiences full employment. Our last task now is to demonstrate that a decline in the

time discount rate  $\rho$  increases both the average growth rate and the average unemployment in our regional economy.

### 3.4. Average growth rate and unemployment

Total output of the consumption good in our regional economy along a BGP is proportional to quality. Mathematically, this means that

$$Y(t, q, \zeta) = Y(q, \zeta) = \frac{x(t/q, \zeta)^{1-\beta} \{q(t)L_E(t, q, \zeta)\}^\beta}{1-\beta} = \frac{q(t)L_E(\zeta)}{1-\beta}. \quad (46)$$

Along the BGP, the number of workers  $L_E(\zeta)$  is given by equations (33) and (34). In other words, the number of production workers changes probabilistically. Now, to describe the average growth rate of our regional economy, note that conditional on being in state  $\zeta$ , the expected growth of the economy along the BGP is given, after some steps of algebra, by

$$g(\zeta) = \frac{[\Lambda(L_R^*)\lambda(1-\varphi) + (1-\Lambda(L_R^*))L_E(E)]L_E(E)}{L_E(\zeta)} - 1. \quad (47)$$

To solve for the average growth rate we need to derive the unconditional probabilities  $p^U$  and  $p^E$  that our regional economy is in states  $U$  and  $E$  respectively. These probabilities  $p^U$  and  $p^E$  have to satisfy the equations  $p^U + p^E = 1$  and  $\Lambda(L_R^*)p^U + \Lambda(L_R^*)p^E = p^U$ . The first of these two equations is the straightforward identity that there are only two states and the second equation follows from the fact that the probability of being in state  $U$  is independent of the current state. This observation tells us that

$$\Lambda(L_R^*) = p^U, \quad 1 - \Lambda(L_R^*) = p^E. \quad (48)$$

Now, using equation (47) the average growth rate in the economy is given by

$$g = p^U g(U) + p^E g(E) = \Lambda(L_R^*) g(U) + \{1 - \Lambda(L_R^*)\} g(E). \quad (49)$$

After some additional steps of algebra, equation (49) can be simplified to

$$g = \left\{ \frac{\Lambda(L_R^*)\varphi}{1-\varphi} + 1 \right\} \{ \Lambda(L_R^*)(\lambda(1-\varphi)-1) + 1 \} - 1. \quad (50)$$

Now, to continue the analysis, let us assume that  $\lambda(1-\varphi) > 1$ . This means that quality improvements are sufficiently large so that the increase in labor productivity  $q$  dominates the effect of having a smaller labor force due to the necessity of retraining. Using this assumption and the fact that  $\Lambda(\cdot)$  is an increasing function, it is clear that  $\partial g / \partial L_R^* > 0$ . Therefore, to analyze the impact of a change in the discount rate on the growth rate of the regional economy, we have to ascertain how the equilibrium number of researchers changes when the discount rate declines.

The allocation of researchers in our regional economy is determined by equation (45) which implies that  $L_R^*$  is decreasing in the time discount rate  $\rho$ . To see this clearly, note that the interest rate is increasing in the time discount rate (see equation (26)). For a given level of researchers  $L_R$ , the LHS of equation (45) is decreasing in  $r$  and the RHS is increasing in  $r$ . Hence, if equation (45) is to be satisfied at higher interest rates, the number of researchers has to decline as the LHS is decreasing in  $L_R$ . This tells us that  $\partial L_R^* / \partial r < 0$  and therefore we deduce that a decline in the time discount rate increases the number of researchers employed and thus the growth rate of our regional economy. To understand this result intuitively, note that as  $\rho$  decreases, the interest rate declines so that profits which materialize in the future are worth more today which is when research expenditures are incurred. This increases the incentive to invest in research which, in turn, increases



our regional economy's growth rate.

Finally, let us focus on the average unemployment rate in the regional economy under study.

Since the unemployment rate depends only on the state of the economy  $\zeta$  and is given by

$$\varphi\{(L-L_R^*)/L\} \text{ if } \zeta=U \Bigg\} = u(\zeta), \quad (51)$$

$$0 \text{ if } \zeta=E$$

using equation (48), the average unemployment rate along the BGP is

$$\bar{u} = p^U u(U) + p^E u(E) = \Lambda(L_R^*) \varphi\left\{\frac{L-L_R^*}{L}\right\}, \quad (52)$$

Partially differentiating the LHS and the RHS of equation (52) with respect to  $L_R^*$  gives us

$$\frac{\partial \bar{u}}{\partial L_R^*} = \frac{\varphi}{L} \{\Lambda'(L_R^*)(L-L_R^*) - \Lambda(L_R^*)\}. \quad (53)$$

We now claim that the last term on the RHS of equation (53) is positive. To see why this claim is true, observe that  $L_R^*$  solves the maximization problem in (42) and that the research firms' maximand is given by  $\kappa \Lambda(L_R^*)(L-L_R^*) - w(t)L_R^*$ , where  $\kappa = 1/(1+r)\{1-\varphi+(1-p^*)/(r+p^*)\}\beta\lambda q$ . Because

$$\frac{\partial}{\partial L_R^*} \{\kappa \Lambda(L_R^*)(L-L_R^*) - w(t)L_R^*\} = 0, \quad (54)$$

it follows that

$$\frac{\partial}{\partial L_R} \{\kappa \Lambda(L_R^*)(L - L_R^*)\} = w(t) > 0. \quad (55)$$

Inspection of equation (55) reveals that  $\Lambda'(L_R^*)(L - L_R^*) - \Lambda(L_R^*) > 0$ .

The above analysis tells us that the average unemployment rate is increasing in the number of researchers employed (see equation (53)). In the above analysis, we have also demonstrated that a decline in the time discount rate will increase the equilibrium number of researchers. Therefore, the average unemployment rate is higher when the time discount rate and hence the equilibrium interest rate declines. From an intuitive standpoint, the reader will note that in our model, there is unemployment only because the use of new technologies require retraining. Because a decline in the time discount rate  $\rho$  leads to a higher probability of innovation, worker retraining occurs more often and this results in a higher equilibrium unemployment rate.

#### 4. Conclusions

In this paper, we used a Schumpeterian model to conduct, to the best of our knowledge for the first time, an explicitly dynamic and stochastic analysis of the ways in which vertical innovations and entrepreneurial activities interact to endogenously generate growth in a regional economy with a single final (consumption) good sector. We first characterized the equilibrium and the BGP for the deterministic version of our model and then we conducted a comparative analysis of the BGP growth rate with the Pareto optimal growth rate for our regional economy. This analysis showed that relative to the BGP allocation, the social planner always employed more labor in R&D, achieved a larger size or quality of innovation, and hence a higher rate of growth.

Next, we studied the stochastic version of our model and this study led to four outcomes. First, we defined the equilibrium and the steady state BGP allocations for our regional economy.

Second, we generalized the notion of the steady state and we ascertained the number of unemployed workers. Third, we showed that our economy experienced bursts of unemployment followed by periods of full employment. Finally, we pointed out that a decline in the interest rate increased the average growth rate and the average unemployment rate in our regional economy.

The analysis in this paper can be extended in a number of directions. Here are two suggestions for extending the research delineated in this paper. First, it would be useful to analyze the innovation/entrepreneurship/growth nexus in a model in which there are multiple open regions that potentially trade with each other. Second, one could study a scenario of the sort studied in this paper but with the key difference that the R&D sector is characterized by the presence of agglomeration effects. Studies of the connections between innovations, entrepreneurial activities, and economic growth that incorporate these features of the problem into the analysis will increase our understanding of the many and varied forces that collectively lead to the development of dynamic and stochastic regional economies.

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